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NGF Interpolation

Newton's Forward Interpolation.

Let $y = f(x)$ denote an unknown function which takes the values $y_0, y_1, y_2, y_3, \ldots y_n$ for the equidistant values $x_0, x_1, x_2, x_3, \ldots x_n$ of the independent variable x. We want to find a polynomial Φ (x) of the n^{th} degree to represent the unknown function. Let this polynomial be written in the form:

$$
\begin{aligned}
\Phi(x) = {} & a_0 \\
& + a_1 (x - x_0) \\
& + a_2 (x - x_0)(x - x_1) \\
& + a_3 (x - x_0)(x - x_1)(x - x_2) \\
& + a_4 (x - x_0)(x - x_1)(x - x_2)(x - x_3) \\
& + a_5 (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4) \\
& \ldots\ldots\ldots\ldots \\
& + a_n (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4) \ldots (x - x_{n-1})
\end{aligned}
$$

Note that if we could determine the coefficients $a_0, a_1, a_2, a_3, \ldots a_n$, in terms of the given y's, then the polynomial $\Phi(x)$ is completely defined.

To do this we rely on the interpolating principle that requires that the unknown polynomial y=f(x) and the interpolating polynomial $\Phi(x)$ must have the same values at the given tabulated values $x_0, x_1, x_2, x_3, \ldots x_n$.

Therefore, at $x=x_0$ both polynomials must have the value y_0.

$$
\begin{aligned}
y_0 = f(x_0) = \Phi(x_0) = {} & a_0 \\
& + a_1\cancelto{0}{(x_0 - x_0)} \\
& + a_2 \cancel{(x_0 - x_0)} (x_0 - x_1) \\
& + a_3 \cancel{(x_0 - x_0)} (x_0 - x_1)(x_0 - x_2) \\
& + a_4 \cancel{(x_0 - x_0)} (x_0 - x_1)(x_0 - x_2)(x_0 - x_3) \\
& + a_5 \cancel{(x_0 - x_0)} (x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4) \\
& \ldots\ldots\ldots\ldots \\
& + a_n \cancel{(x_0 - x_0)} (x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4) \ldots (x_0 - x_{n-1})
\end{aligned}
$$

$\therefore\ a_0 = y_0$

By the same token, at $x=x_1$ both polynomials must have the value y_1.

$$
\begin{aligned}
y_1 = f(x_1) = \Phi(x_1) = {} & a_0 \\
& + a_1(x_1 - x_0) \\
& + a_2 (x_1 - x_0)\cancelto{0}{(x_1 - x_1)}
\end{aligned}
$$

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$$
\begin{aligned}
&+\ a_3 (x_1 - x_0)\,\cancel{(x_1 - x_1)}\,(x_1 - x_2)\\
&+\ a_4 (x_1 - x_0)\,\cancel{(x_1 - x_1)}\,(x_1 - x_2)\,(x_1 - x_3)\\
&\qquad\qquad\ldots\ldots\ldots\ldots\\
&+\ a_n (x_1 - x_0)\,\cancel{(x_1 - x_1)}\,(x_1 - x_2)\,(x_1 - x_3)\ (x_1 - x_4)\ \ldots.(x_1 - x_{n-1})
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ y_1 = f(x_1) = \Phi(x_1) &= a_0 + a_1(x_1 - x_0)\\
&= a_0 + a_1(x_1 - x_0)\\
&= y_0 + a_1(x_1 - x_0)
\end{aligned}
$$

$$y_1 - y_0 = a_1(x_1 - x_0)$$

$$\therefore\quad a_1 = (y_1 - y_0)\ /(x_1 - x_0),$$

Since $\Delta y_0 = (y_1 - y_0)$ let $h = (x_1 - x_0)$,

$$\therefore a_1 = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y_0}{h}$$

At $x=x_2$ both polynomials must have the value y_1.

$$
\begin{aligned}
y_2 = f(y_2) = \Phi(x_2) = &\ a_0\\
&+\ a_1(x_2 - x_0)\\
&+\ a_2 (x_2 - x_0)\,(x_2 - x_1)\\
&+\ a_3 (x_2 - x_0)\,(x_2 - x_1)\,\cancelto{0}{(x_2 - x_2)}
\end{aligned}
$$

$$y_2 = a_0 + a_1(x_2 - x_0) + a_2 (x_2 - x_0)\,(x_2 - x_1)$$

$$\therefore y_2 = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

$$\therefore y_2 = y_0 + \frac{y_1 - y_0}{h}(2h) + a_2(2h)(h)$$

$$
\begin{aligned}
&\therefore y_2 = y_0 + (y_1 - y_0)(2) + a_2(2h)(h)\\
&\quad a_2(2h)(h) = y_2 - y_0 - (2)(y_1 - y_0)\\
&\quad a_2(2h)(h) = y_2 - y_0 - 2y_1 + 2y_0)\\
&\quad a_2(2h)(h) = y_0 - 2y_1 + y_2\\
&\therefore a_2 = \frac{y_0 - 2y_1 + y_2}{2h^2} = \frac{\Delta^2 y_0}{2h^2}
\end{aligned}
$$

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Repeating the process:

$$\therefore a_0 = y_0, \quad a_1 = \frac{\Delta y_0}{h}, \quad a_2 = \frac{\Delta^2 y_0}{2h^2}, \quad a_3 = \frac{\Delta^3 y_0}{6h^3}, \quad a_4 = \frac{\Delta^4 y_0}{24h^4}, \quad a_5 = \frac{\Delta^5 y_0}{120h^5}, \ldots$$

or

$$\therefore a_0 = y_0, \quad a_1 = \frac{\Delta y_0}{h}, \quad a_2 = \frac{\Delta^2 y_0}{2!h^2}, \quad a_3 = \frac{\Delta^3 y_0}{3!h^3}, \quad a_4 = \frac{\Delta^4 y_0}{4!h^4}, \quad a_5 = \frac{\Delta^5 y_0}{5!h^5}, \ldots$$

Substituting in the $\Phi(x)$:

$$\begin{aligned}
\Phi(x) = y_0 & \\
&+ \frac{\Delta y_0}{h}(x - x_0) \\
&+ \frac{\Delta^2 y_0}{2!h^2}(x - x_0)(x - x_1) \\
&+ \frac{\Delta^3 y_0}{3!h^3}(x - x_0)(x - x_1)(x - x_2) \\
&+ \frac{\Delta^4 y_0}{4!h^4}(x - x_0)(x - x_1)(x - x_2)(x - x_3) \\
&+ \ldots\ldots \\
&+ \frac{\Delta^n y_0}{n!h^n}(x - x_0)(x - x_1)(x - x_2)(x - x_3) \ldots (x - x_{n-1})
\end{aligned}$$

Distributing the h:

$$\begin{aligned}
\Phi(x) = y_0 \quad &+ \Delta y_0 \frac{(x - x_0)}{h} \\
&+ \frac{\Delta^2 y_0}{2!} \frac{(x - x_0)}{h} \frac{(x - x_1)}{h} \\
&+ \frac{\Delta^3 y_0}{3!} \frac{(x - x_0)}{h} \frac{(x - x_1)}{h} \frac{(x - x_2)}{h}
\end{aligned}$$

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$$+\frac{\Delta^4 y_0}{4!}\frac{(x-x_0)}{h}\frac{(x-x_1)}{h}\frac{(x-x_2)}{h}\frac{(x-x_3)}{h}$$

$$+\ldots\ldots$$
$$+\frac{\Delta^n y_0}{n!}\frac{(x-x_0)}{h}\frac{(x-x_1)}{h}\frac{(x-x_2)}{h}\frac{(x-x_3)}{h}\ldots\frac{(x-x_{n-1})}{h}$$

$$Let \quad u=\frac{(x-x_0)}{h},$$
$$then \quad \frac{(x-x_1)}{h}=\frac{(x-(x_0+h))}{h}=\frac{(x-x_0-h))}{h}=\frac{(x-x_0)}{h}-\frac{h}{h}=u-1$$
$$\frac{(x-x_2)}{h}=u-2, \quad \frac{(x-x_3)}{h}=u-3, \ldots..$$

Substituting:

$$\Phi(x)=y_0+\frac{\Delta y_0}{1!}u+\frac{\Delta^2 y_0}{2!}u(u-1)+\frac{\Delta^3 y_0}{3!}u(u-1)(u-2)+\ldots$$

Or

$$\Phi(x)=y_0+\Delta y_0\frac{u}{1}+\Delta^2 y_0\frac{u(u-1)}{2!}+\Delta^3 y_0\frac{u(u-1)(u-2)}{3!}+\ldots$$

But,

$$\binom{u}{2}=\frac{u!}{2!\{u-2\}!}=\frac{u(u-1)(u-2)(u-3)\ldots 1}{2[(u-2)(u-3)(u-4)\ldots 1]}=\frac{u(u-1)}{2}$$

$$\therefore \quad \Phi(x)=y_0+\binom{u}{1}\Delta y_0+\binom{u}{2}\Delta^2 y_0+\binom{u}{3}\Delta^3 y_0+\ldots$$

Sir Isaac Newton.

English physicist and mathematician who was born into a poor farming family. Luckily for humanity, Newton was not a good farmer, and was sent to Cambridge to study to become a preacher. At Cambridge, Newton studied mathematics, being especially strongly influenced by Euclid, although he was also influenced by Baconian and Cartesian philosophies. Newton was forced to leave Cambridge when it was closed because of the plague, and it was during this period that he made some of his most significant discoveries. With the reticence he was to show later in life, Newton did not, however, publish his results.

Newton invented a scientific method, which was truly universal in its scope. Newton presented his methodology as a set of four rules for scientific reasoning. These rules were stated in the *Principia* and proposed that (1) we are to admit no more causes of natural things such as are both true and sufficient to explain their appearances, (2) the same natural effects must be assigned to the same causes, (3) qualities of bodies are to be esteemed as universal, and (4) propositions deduced from observation of phenomena should be viewed as accurate until other phenomena contradict them.