



M42- TRIGONOMETRY

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$$\sin(\theta) = \frac{b}{c} \Leftrightarrow \csc = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{a}{c} \Leftrightarrow \sec = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{b}{a} \Leftrightarrow \cot = \frac{1}{\tan(\theta)}$$

$$\sin(-x) = -\sin(x)$$

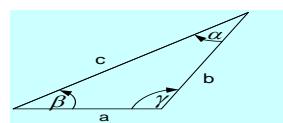
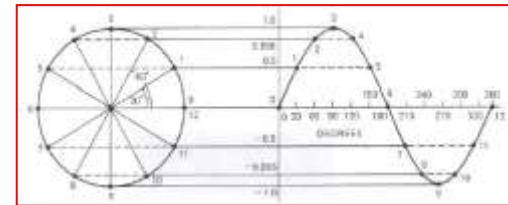
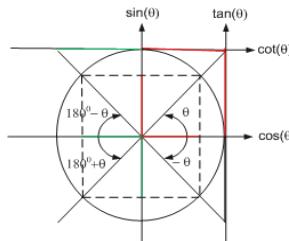
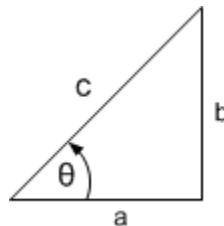
$$\sin^2(x) + \cos^2(x) = 1$$

$$\cos(-x) = \cos(x)$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\tan(-x) = -\tan(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$



$$\overbrace{\text{Arc Length : } s = r\theta_{rad}}^{\theta \text{ in rads}}$$

$$\text{Sector Area : } \frac{1}{2} r^2 \theta_{rad}$$

$$v = \frac{s}{t}, \quad \omega = \frac{\theta_{rad}}{t}$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

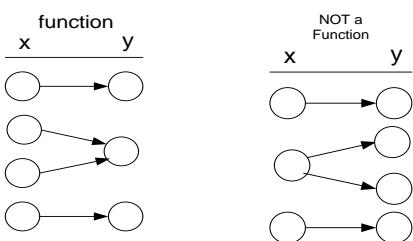
$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\text{if } p = \frac{(a+b+c)}{2}, \text{ then}$$

$$A = \sqrt{p(p-a)(p-b)(p-c)}$$

Function	Domain	Range
$\sin t$	$\{ t -\infty < t < \infty \}$	$\{ y -1 \leq y \leq 1 \}$
$\cos t$	$\{ t -\infty < t < \infty \}$	$\{ y -1 \leq y \leq 1 \}$
$\tan t$	$\{ t -\infty < t < \infty, t \neq (2n+1)\pi/2 \}$	$\{ y -\infty < y < \infty \}$
$\csc t$	$\{ t -\infty < t < \infty, t \neq n\pi \}$	$\{ y y \geq 1, y \leq -1 \}$
$\sec t$	$\{ t -\infty < t < \infty, t \neq (2n+1)\pi/2 \}$	$\{ y y \geq 1, y \leq -1 \}$
$\cot t$	$\{ t -\infty < t < \infty, t \neq n\pi \}$	$\{ y -\infty < y < \infty \}$

Function	Period	Domain
$\sin t$	2π	$0, 2\pi$
$\cos t$	2π	$0, 2\pi$
$\tan t$	π	$-\pi/2, \pi/2$
$\cot t$	π	$0, \pi$



θ (degrees)	0°	30°	45°	60°	90°	180°	270°
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undef.	0	Undef.



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$|a| = \text{Amplitude}, \quad b > 0, \quad c = \text{Phase Shift}, \quad d = \text{Vertical translation}, \quad p = \text{Period}$

$$y = d + a \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (bx + c)$$

$$d = \begin{cases} + \uparrow \\ - \downarrow \end{cases}$$

$$|a| = \begin{cases} > 1 \uparrow \\ < 1 \downarrow \end{cases}$$

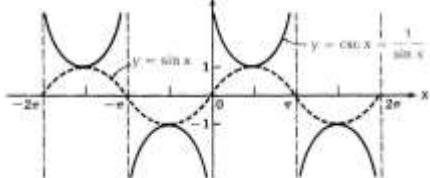
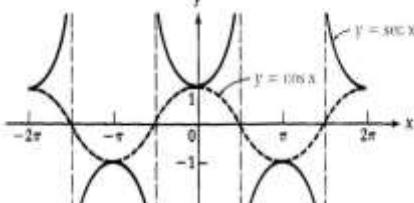
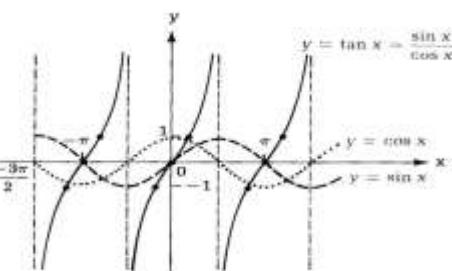
$$p = \frac{2\pi}{b} \quad \begin{cases} 0 < b < 1 \leftarrow \rightarrow \\ b > 1 \rightarrow \leftarrow \end{cases}$$

Period

Bounds

$$\underbrace{(bx + c) = 0}_{x_1 = -\frac{c}{b}} \quad \underbrace{(bx + c) = 2\pi}_{x_2 = \frac{2\pi}{b} - \frac{c}{b}}$$

$$c = -\frac{c}{b} \quad \begin{cases} < 0 \leftarrow L \\ > 0 \rightarrow R \end{cases}$$



$$y = d + a \begin{Bmatrix} \tan \\ \cot \end{Bmatrix} (bx + c)$$

$$d = \begin{cases} + \uparrow \\ - \downarrow \end{cases}$$

$$|a| = \begin{cases} \text{Meaningless} \\ -\infty \leftarrow \rightarrow +\infty \end{cases}$$

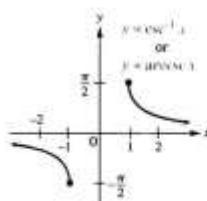
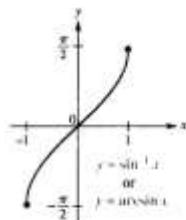
$$p = \frac{\pi}{b}$$

Asymptotes \Rightarrow (Period Bounds)

$$\tan : \underbrace{(bx + c) = -\frac{\pi}{2}}_{x_1 = -\frac{\pi}{b} - \frac{c}{b}} \quad \underbrace{(bx + c) = \frac{\pi}{2}}_{x_2 = \frac{\pi}{b} - \frac{c}{b}}$$

$$\cot : \underbrace{(bx + c) = 0}_{x_1 = -\frac{c}{b}} \quad \underbrace{(bx + c) = \pi}_{x_2 = \frac{\pi}{b} - \frac{c}{b}}$$

$$c = -\frac{c}{b} \quad \begin{cases} < 0 \leftarrow L \\ > 0 \rightarrow R \end{cases}$$



$$y = d + a \begin{Bmatrix} \sec \left(\frac{1}{\cos} \right) \\ \csc \left(\frac{1}{\sin} \right) \end{Bmatrix} (bx + c)$$

*Graph $\begin{Bmatrix} \sin(bx + c) \\ \cos(bx + c) \end{Bmatrix}$ first,
then the
sec or csc.*

Simple Harmonic Motion

$$y = \begin{Bmatrix} a \sin(\omega t) \\ a \cos(\omega t) \end{Bmatrix}$$

$|a| = \text{Aplitude}$

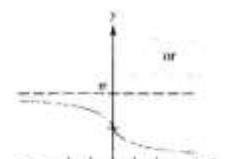
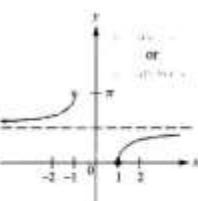
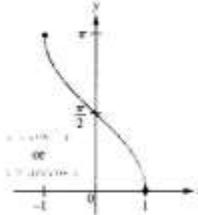
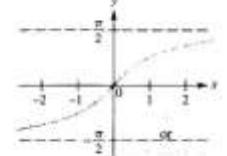
$$\text{Period} : T = \frac{2\pi}{\omega}$$

$$\text{Frequency} : f = \frac{\omega}{2\pi}, \text{ in Hz}$$

$\omega t = \text{rad or } \theta^0$

$$\omega = \text{Angular speed} = \frac{\theta}{t}, \text{ rad/s.}$$

$\dots \rightarrow \text{Linear speed} \cdot s = r\theta = r\omega$





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$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \operatorname{ctn}^2 \theta = \csc^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\tan(\theta \pm \phi) = \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \tan \phi}$$

$$\sin \theta + \sin \phi = 2 \sin \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi)$$

$$\sin \theta - \sin \phi = 2 \cos \frac{1}{2}(\theta + \phi) \sin \frac{1}{2}(\theta - \phi)$$

$$\cos \theta + \cos \phi = 2 \cos \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi)$$

$$\cos \theta - \cos \phi = -2 \sin \frac{1}{2}(\theta + \phi) \sin \frac{1}{2}(\theta - \phi)$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\sin n\theta = 2 \sin(n-1)\theta \cos \theta - \sin(n-2)\theta$$

$$\cos n\theta = 2 \cos(n-1)\theta \cos \theta - \cos(n-2)\theta$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin \theta \cos \phi = \frac{1}{2} \sin(\theta + \phi) + \frac{1}{2} \sin(\theta - \phi)$$

$$\sin \theta \sin \phi = \frac{1}{2} \cos(\theta - \phi) - \frac{1}{2} \cos(\theta + \phi)$$

$$\cos \theta \cos \phi = \frac{1}{2} \cos(\theta - \phi) + \frac{1}{2} \cos(\theta + \phi)$$

$$\sin^3 \theta = \frac{1}{4}(3 \sin \theta - \sin 3\theta)$$

$$\cos^3 \theta = \frac{1}{4}(\cos 3\theta + 3 \cos \theta)$$

